

Searching for gravitational waves from Cassiopeia A with LIGO

K Wette³, B J Owen⁷, B Allen^{2,13}, M Ashley³,
 J Betzwieser⁵, N Christensen⁴, T D Creighton⁸,
 V Dergachev¹¹, I Gholami¹, E Goetz¹¹, R Gustafson¹¹,
 D Hammer¹³, D I Jones¹², B Krishnan¹, M Landry⁶,
 B Machenschalk², D E McClelland³, G Mendell⁶,
 C J Messenger², M A Papa^{1,13}, P Patel⁵, M Pitkin¹⁰,
 H J Pletsch², R Prix², K Riles¹¹, L Sancho de la Jordana⁹,
 S M Scott³, A M Sintes^{9,1}, M Trias⁹, J T Whelan¹ and
 G Woan¹⁰

¹Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-14476 Golm, Germany

²Albert-Einstein-Institut, Max-Planck-Institut für Gravitationsphysik, D-30167 Hannover, Germany

³Australian National University, Canberra, 0200, Australia

⁴Carleton College, Northfield, MN 55057, USA

⁵LIGO - California Institute of Technology, Pasadena, CA 91125, USA

⁶LIGO Hanford Observatory, Richland, WA 99352, USA

⁷The Pennsylvania State University, University Park, PA 16802, USA

⁸The University of Texas at Brownsville and Texas Southmost College, Brownsville, TX 78520, USA

⁹Universitat de les Illes Balears, E-07122 Palma de Mallorca, Spain

¹⁰University of Glasgow, Glasgow, G12 8QQ, United Kingdom

¹¹University of Michigan, Ann Arbor, MI 48109, USA

¹²University of Southampton, Southampton, SO17 1BJ, United Kingdom

¹³University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA

E-mail: karl.wette@ligo.org

Abstract. We describe a search underway for periodic gravitational waves from the central compact object in the supernova remnant Cassiopeia A. The object is the youngest likely neutron star in the Galaxy. Its position is well known, but the object does not pulse in any electromagnetic radiation band and thus presents a challenge in searching the parameter space of frequency and frequency derivatives. We estimate that a fully coherent search can, with a reasonable amount of time on a computing cluster, achieve a sensitivity at which it is theoretically possible (though not likely) to observe a signal even with the initial LIGO noise spectrum. Cassiopeia A is only the second object after the Crab pulsar for which this is true. The search method described here can also obtain interesting results for similar objects with current LIGO sensitivity.

PACS numbers: 04.80.Nn, 97.60.Bw, 97.60.Jd

1. Introduction

The LIGO Scientific Collaboration (LSC) has so far published three types of searches for periodic gravitational waves (GWs): searches for known non-accreting pulsars [1, 2, 3, 4], for the non-pulsing low-mass X-ray binary Sco X-1 [5, 6], and all-sky searches for as yet unknown neutron stars [5, 7, 8, 9]. The first and last types of search are approaching the indirect upper limits on gravitational wave emission inferred from the observed spindowns (spin frequency derivatives) of pulsars and supernova-based estimates of the neutron star population of the galaxy [5].

Here we discuss the first of a fourth type of search for periodic gravitational waves: directed searches, which target likely neutron stars whose sky position is known to high accuracy, but whose spin frequencies and frequency evolution are not known at all. We describe such a search, which is currently underway, directed at the central compact object in the supernova remnant Cassiopeia A (Cas A). The data analysis challenge is to search a large parameter space of possible frequencies and frequency evolutions. We describe the object, estimate the computational costs of the search, and show that when the search of data from LIGO's recently completed S5 run is completed, it will beat the indirect limit on GW strain for Cas A. We also indicate how cost and sensitivity estimates can be extended to other directed searches.

2. The central compact object in Cas A

Cas A is a core-collapse supernova remnant, currently the youngest known in the Galaxy [10]. A central X-ray point source was discovered in first-light images taken by the Chandra X-Ray Observatory, indicating the presence of a central compact object (CCO). The nature of the CCO remains uncertain. No radio pulsations or γ -ray emission have been observed, and there is no pulsar wind nebula observed in X-ray or radio; it is unlikely therefore that the CCO is an active pulsar [11]. Proposed explanations include that it might be a young radio-quiet neutron star, or an accretion disk associated with a neutron star or black hole, or that it might be related to a type of slowly rotating neutron star known as an anomalous X-ray pulsar (AXP) or a soft γ -ray repeater (SGR) [11, 12]. Only in the first scenario could GW emission be detectable by LIGO. What makes Cas A an attractive target is its youth: the stars with the highest indirect limits (see next section) on gravitational radiation are young, and one could argue on theoretical grounds that any deformations left over from the violent birth of the star have had less time to be smoothed away by mechanisms such as viscoelastic creep. Young stars also spin more quickly than old ones. Of the seven confirmed CCOs, only two (possibly three) have measured spin periods [13]. The fastest is radiating gravitational waves at 20 Hz, just below the LIGO frequency band, but the other CCOs are also much older than Cas A.

For the purpose of a directed search, we need to know the object's right ascension and declination. Chandra observations [11] have obtained these to sub-arcsecond accuracy [$\alpha = 23^{\text{h}}23^{\text{m}}(27.945 \pm 0.05)^{\text{s}}$, $\delta = 58^{\circ}48'(42.51 \pm 0.4)''$], which is sufficient for any GW observation. In order to define the range of search parameters and give an indirect limit on GW emission from the object, we also need the distance, age, and moment of inertia. The distance to Cas A has been estimated from the radial velocities of knots of ejected material to be $3.4_{-0.1}^{+0.3}$ kpc [14]. Extrapolation of the proper motions of outer ejecta knots suggest a convergence date of 1681 ± 19 , consistent with a possible observation by John Flamsteed in 1680 [10]. Since computational costs are higher for

younger objects, we play it safe by taking 300 years (the approximate lower bound) as our fiducial age estimate. In what follows we use the canonical neutron star moment of inertia of 10^{45} g cm^2 , although modern equations of state predict values higher for most neutron stars by a factor 2 or 3 [15].

3. Indirect limits

Indirect limits on the gravitational wave emission from rotating neutron stars are found by assuming that the gravitational wave luminosity is bounded by the time derivative of the total rotational kinetic energy:

$$\left(\frac{dE}{dt}\right)_{\text{gw}} = \frac{32G}{5c^5} I_{zz}^2 \epsilon^2 (\pi f)^6 \leq -\frac{d}{dt} \left(\frac{1}{2} \pi^2 I_{zz} f^2 \right) = -\left(\frac{dE}{dt}\right)_{\text{rot}}, \quad (1)$$

where ϵ is the equatorial ellipticity, I_{zz} the principal moment of inertia (assumed constant), and f the gravitational wave frequency (assumed to be twice the spin frequency) [5, 16]. This condition is rearranged to give the “spindown” upper bounds on the ellipticity and the GW strain tensor amplitude h_0 :

$$\epsilon \leq \sqrt{\frac{5c^5}{32\pi^4 G I_{zz}} \frac{-\dot{f}}{f^5}}, \quad h_0 \leq \frac{1}{D} \sqrt{\frac{5G I_{zz}}{2c^3} \frac{-\dot{f}}{f}}, \quad (2)$$

The second limit is found from the first by substituting

$$h_0 = \frac{4\pi^2 G}{c^4} \frac{I_{zz} \epsilon f^2}{D}. \quad (3)$$

where D is the distance of the source [5, 17].

For a directed search, the GW frequency f and its time derivative \dot{f} are unknown, but the age is known. If we assume that the star is spinning down with $\dot{f} \propto f^n$, and that it is currently spinning significantly more slowly than it was at birth, we can relate the frequency evolution to the characteristic age τ and braking index n by [18, 19, 5]

$$\tau \approx \frac{1}{n-1} \left(\frac{f}{-\dot{f}} \right), \quad n = \frac{f\ddot{f}}{\dot{f}^2}. \quad (4)$$

If the spindown is dominated by GW from a constant mass quadrupole, then $n = 5$ and τ is the true age of the star. Substituting into the spindown limits (2) gives

$$\epsilon_{\text{age}} \leq \sqrt{\frac{5c^5}{128\pi^4 G I_{zz} \tau f^4}}, \quad h_{\text{age}} \leq \frac{1}{D} \sqrt{\frac{5G I_{zz}}{8c^3 \tau}}. \quad (5)$$

Using the numbers for Cas A from the previous section we get

$$h_{\text{age}} \leq 1.2 \times 10^{-24} \left(\frac{3.4 \text{ kpc}}{D} \right) \sqrt{\left(\frac{I_{zz}}{10^{45} \text{ g cm}^2} \right) \left(\frac{300 \text{ years}}{\tau} \right)} \quad (6)$$

$$\epsilon_{\text{age}} \leq 3.9 \times 10^{-4} \left(\frac{100 \text{ Hz}}{f} \right)^2 \sqrt{\left(\frac{10^{45} \text{ g cm}^2}{I_{zz}} \right) \left(\frac{300 \text{ years}}{\tau} \right)}. \quad (7)$$

Below we will consider searches over the range $n = 2-7$, including the possibility that n has changed since the supernova and thus a lifetime-averaged value is appropriate. Considering this, the uncertainty in D , and the fact that I_{zz} may be triple our fiducial value (see discussion in [3]), these fiducial indirect upper limits are uncertain by about a factor of 2. Some theories of quark matter allow for ellipticities in the range indicated,

though normal neutron star models do not [20, 21, 22]. An internal magnetic field of order 10^{16} G could also produce such ellipticities [23, 24, 25, 26], although it is not clear if such a field is stable, and if the external field is this strong then the star by now has spun down out of the LIGO frequency band. The age-based indirect limits serve, like the spindown limits, as indicators of which objects are interesting, but since they are based on less information they are not as solid as the spindown limits. It is not known if Cas A spins in the LIGO band (period ≤ 50 ms), and indeed only 10% of known pulsars do so [27]. Thus a search such as we describe could detect an object on the speculative end of the range of theoretical predictions.

4. Search method

The LSC uses both fully coherent [1, 2, 3, 4, 5] and semi-coherent [6, 7, 8, 9] methods to search for periodic gravitational waves. Semi-coherent methods are computationally cheaper than coherent methods, but coherent methods can achieve greater sensitivity if the cost is feasible.

For a young neutron star such as Cas A the integration time needed is short enough (see next section) for us to pursue enhanced sensitivity without undue computational cost. We therefore use the fully coherent \mathcal{F} -statistic search [5], as implemented by the `ComputeFStatistic.v2` routine in the LSC Algorithm Library [28]. This routine computes optimal filters for the gravitational wave signal, including modulation by the detector beam patterns, in multiple interferometers which are treated as a coherent network [17, 29]. This search uses data from the 4km LIGO interferometers at Hanford, WA, and Livingston, LA.

The computation is conducted in the frequency domain using short Fourier transforms (SFTs) of segments of strain data, typically of 30 minutes duration so that the GW frequency will remain in one frequency bin over the length of the SFT [5]. The SFTs are vetoed by a suite of data quality flags to remove poorer quality data. For windows of up to 15 days during the first year of the S5 run the duty cycle – the ratio of post-veto SFT live time to total time span, averaged over interferometers – can somewhat exceed 70%.

A search for a young neutron star such as Cas A, which is younger than objects considered in previous LIGO multi-template searches, must cover a greater spindown parameter space including a second frequency derivative (see next section). This has required the extension of existing LSC software to efficiently cover a three-dimensional space using the parameter space metric. The points are distributed on a body-centered cubic (bcc or A_3^*) lattice, which is known to be the optimal lattice covering in three dimensions [30].

In the event no plausible signal is found, we will set upper limits by methods similar to the frequentist analyses in [1, 5]. These are based on Monte Carlo simulations searching the data for a multitude of software-injected signals with a distribution of amplitudes, inclination angles, and polarization angles in each frequency bin. We will also test on a smaller set of simulated signals which were hardware injected into the S5 data.

5. Estimated cost and sensitivity

The sensitivity of a search for periodic signals can be put in terms of the 95% confidence limit on GW strain tensor amplitude, which takes the form

$$h_0^{95\%} = \Theta \sqrt{S_h(f)/T_{\text{dat}}}. \quad (8)$$

Here S_h is the strain noise power spectral density, T_{dat} is the data live time, and Θ is a statistical threshold factor which depends on the parameter space and other details of the data analysis pipeline. For a coherent multi-interferometer search, the limits add in inverse quadrature. Monte Carlo simulations searching for injected signals from Cas A, as well as the results of the similar multi-template Crab search [4], indicate that Θ is in the mid-30s for a directed search, and thus we use 35 in our estimates below. Because Θ is determined by the tail of a Gaussian distribution, it is very weakly dependent on the volume of parameter space searched. However the data live time T_{dat} is computationally limited and thus does depend on the parameter space.

The parameter space range is chosen as follows. The frequency band is chosen to be 100–300 Hz, which surrounds the band where the LIGO interferometers are most sensitive. As we shall see below, this is roughly the band over which a directed search can beat the indirect limit on h_0 with reasonable computational cost. The frequency derivative ranges are chosen based on considering braking indices n in the range 2–7. This range covers all known pulsars, except the Vela pulsar which is visibly interacting with its wind nebula (nonexistent for Cas A). It also includes the values for radiation dominated by a static dipole or quadrupole ($n = 3$ or 5) as well as a saturated r -mode ($n = 7$) [31]. Thus the range of each frequency derivative depends on the lower derivatives, and we have

$$100 \text{ Hz} \leq f \leq 300 \text{ Hz}, \quad \frac{f}{6\tau} \leq -\dot{f} \leq \frac{f}{\tau}, \quad \frac{2\dot{f}^2}{f} \leq \ddot{f} \leq \frac{7\dot{f}^2}{f}. \quad (9)$$

Note that the range of \ddot{f} by definition is related to the present-day braking index, while the range of \dot{f} corresponds to an average braking index over the lifetime of the star. Thus we allow for the braking index varying over time between the indicated limits.

There remains the problem of efficiently tiling, or choosing specific points in parameter space for which to compute the \mathcal{F} -statistic. It is straightforward to apply the method of [32] to find the parameter space metric [33]

$$\gamma_{jk} = \frac{4\pi^2 T_{\text{span}}^{j+k+2} (j+1)(k+1)}{(j+2)!(k+2)!(j+k+3)}, \quad (10)$$

where the components are with respect to the k th derivative of the GW frequency at the beginning of the observation, T_{span} is the total duration of data (including dropouts), and the indices j, k take integer values between 0 and the highest derivative considered (2 for Cas A). This metric, which is the Fisher information matrix with a phase constant projected out, is used to set up an efficient tiling which takes advantage of the covariances between parameters. The number of points needed for an optimal (bcc or A_3^*) tiling is given by [34]

$$N_p \simeq 0.19 \mu^{-3/2} \sqrt{\det \gamma} \frac{f_{\text{max}}^3}{\tau^3}, \quad (11)$$

where μ is the mismatch and we have performed the integral in equation (24) of [34] using the ranges (9) and discarding the lower bound on frequency, which is only a few

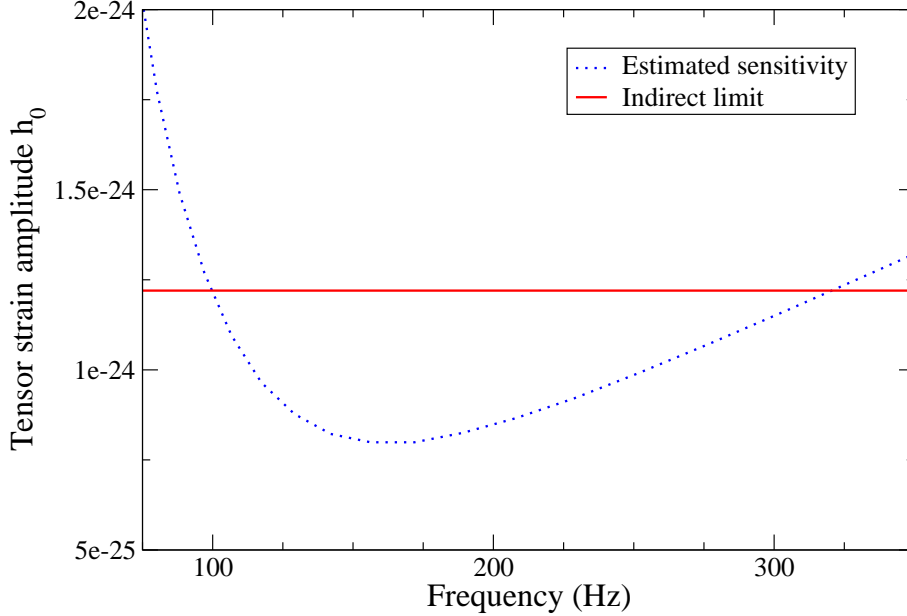


Figure 1. Estimated sensitivity of an S5 search compared to the indirect limit on GW emission for Cas A. Search parameters are the fiducial ones described in the text.

percent correction. We determine the highest frequency derivative needed by finding k such that $\gamma_{kk}\Delta_k^2 > \mu$, where Δ_k is the range of the k th frequency derivative and we take μ to be 20% (typical for periodic signal searches). In our case \ddot{f} is required for T_{span} greater than about a week; as shown below, this applies for any search competitive with the indirect limit.

Since equation (11) is obtained by dividing the proper volume of the parameter space by the proper volume per template, we expect it to underestimate N_p of a practical implementation due to the need to cover the edges of the parameter space. Because the extent of our parameter space in \ddot{f} is often comparable to or less than the unit cell length of a single template, we expect that an ideal lattice covering would require several times the ideal number of templates in (11). Technical limitations of a speedy—and therefore simple—template bank generation algorithm also require us to lay extra templates to guarantee that the edges of the parameter space are completely covered. We have found from Monte Carlo simulations that the combination of these effects can cause (11) to underestimate N_p by up to an order of magnitude. Even in this worst case, without any improvement of existing template bank algorithms, the computational cost is still feasible since our fiducial estimate below is for a small number of computing nodes. The size of the template bank should not significantly affect the upper limits, which are very weakly dependent on the number of templates and thus on the number of statistical trials.

Finally we estimate the computational cost and sensitivity of a directed search. Preliminary runs on nodes of the APAC cluster [35] find a timing of about 6×10^{-7} s

per template per SFT. Assuming 30-minute SFTs and two interferometers with 70% duty cycle, the computing time for the search (exclusive of Monte Carlo simulations to compute upper limits) is

$$20 \text{ days} \left(\frac{f_{\max}}{300 \text{ Hz}} \right)^3 \left(\frac{300 \text{ years}}{\tau} \right)^3 \left(\frac{T_{\text{span}}}{12 \text{ days}} \right)^7 \left(\frac{200}{\text{nodes}} \right). \quad (12)$$

For these fiducial parameters and two interferometers with the initial LIGO design noise spectrum [36] and 70% duty cycle, the sensitivity curve (8) is plotted in figure 1. The minimum of the curve (smallest detectable h_0) is

$$8.0 \times 10^{-25} \left(\frac{12 \text{ days}}{T_{\text{span}}} \right)^{-1/2} \quad \text{or} \quad 8.0 \times 10^{-25} \left(\frac{f_{\max}}{300 \text{ Hz}} \right)^{3/14} \left(\frac{300 \text{ years}}{\tau} \right)^{3/14}, \quad (13)$$

where the latter scalings allow T_{span} to vary at fixed computational cost and are useful for evaluating searches for other objects. Combining the previous two equations indicates that the sensitivity only improves as the 14th root of the computational cost, and thus there is not much point in integrating for significantly longer without an improved semi-coherent analysis method.

Thus we see that this search for Cas A, when completed on S5 data, will beat the fiducial indirect limit on GW emission from about 100 to 300 Hz. This will double the number of objects (after the Crab pulsar) for which initial LIGO has beaten an indirect limit. Similar searches can be made for other suspected young neutron stars.

Acknowledgments

We are grateful to the LSC internal reviewers for helpful comments on the manuscript. This work was supported by Australian Research Council grant DP-0770426 and USA National Science Foundation grants PHY-0245649, PHY-0555628, and cooperative agreement PHY-0114375 (the Penn State Center for Gravitational Wave Physics). This article has been assigned LIGO document number P070123-02-Z.

References

- [1] B. Abbott et al. Setting upper limits on the strength of periodic gravitational waves from PSR J1939+2134 using the first science data from the GEO 600 and LIGO detectors. *Phys. Rev. D*, 69:082004, 2004.
- [2] B. Abbott et al. Limits on gravitational wave emission from selected pulsars using LIGO data. *Phys. Rev. Lett.*, 94:181103, 2005.
- [3] B. Abbott et al. Upper limits on gravitational wave emission from 78 radio pulsars. *Phys. Rev. D*, 76:042001, 2007.
- [4] B. Abbott et al. Beating the spin-down limit on gravitational wave emission from the Crab pulsar. *Astrophys. J.*, 683:L45, 2008.
- [5] B. Abbott et al. Coherent searches for periodic gravitational waves from unknown isolated sources and Scorpius X-1: results from the second LIGO science run. *Phys. Rev. D*, 76:082001, 2007.
- [6] B. Abbott et al. Upper limit map of a background of gravitational waves. *Phys. Rev. D*, 76:082003, 2007.
- [7] B. Abbott et al. First all-sky upper limits from LIGO on the strength of periodic gravitational waves using the Hough transform. *Phys. Rev. D*, 72:102004, 2005.
- [8] B. Abbott et al. All-sky search for periodic gravitational waves in LIGO S4 data. *Phys. Rev. D*, 77:022001, 2008.
- [9] B. Abbott et al. The Einstein@Home search for periodic gravitational waves in LIGO S4 data. *Phys. Rev. D* accepted, 2008.

- [10] R. A. Fesen, M. C. Hammell, J. Morse, R. A. Chevalier, K. J. Borkowski, M. A. Dopita, C. L. Gererdy, S. S. Lawrence, J. C. Raymond, and S. van den Bergh. The expansion asymmetry and age of the Cassiopeia A supernova remnant. *Astrophys. J.*, 645:283, 2006.
- [11] R. A. Fesen, G. G. Pavlov, and D. Sanwal. Near-infrared and optical limits for the central X-ray point source in the Cassiopeia A supernova remnant. *Astrophys. J.*, 636:848, 2006.
- [12] D. Chakrabarty, M. J. Pivovarov, L. E. Hernquist, J. S. Heyl, and R. Narayan. The central X-ray point source in Cassiopeia A. *Astrophys. J.*, 548:800, 2001.
- [13] Andrea De Luca. Central Compact Objects in Supernova Remnants. *AIP Conf. Proc.*, 983:311–319, 2008.
- [14] J. E. Reed, J. J. Hester, A. C. Fabian, and P. F. Winkler. The three-dimensional structure of the Cassiopeia A supernova remnant. I. The spherical shell. *Astrophys. J.*, 440:706, 1995.
- [15] Michal Bejger, T. Bulik, and P. Haensel. Constraints on the dense matter EOS from the measurements of PSR J0737-3039A moment of inertia and PSR J0751+1807 mass. *Mon. Not. R. Astron. Soc.*, 364:635, 2005.
- [16] Mark Zimmermann and Eugene Szedenits. Gravitational waves from rotating and precessing rigid bodies: Simple models and applications to pulsars. *Phys. Rev. D*, 20:351, 1979.
- [17] P. Jaranowski, A. Królak, and B. F. Schutz. Data analysis of gravitational-wave signals from spinning neutron stars. I. The signal and its detection. *Phys. Rev. D*, 58:063001, 1998.
- [18] J. P. Ostriker and J. E. Gunn. On the nature of pulsars. I. Theory. *Astrophys. J.*, 157:1395, 1969.
- [19] C. Palomba. Pulsars ellipticity revised. *Astron. Astrophys.*, 354:163, 2000.
- [20] Benjamin J. Owen. Maximum elastic deformations of compact stars with exotic equations of state. *Phys. Rev. Lett.*, 95:211101, 2005.
- [21] Lap-Ming Lin. Constraining crystalline color superconducting quark matter with gravitational-wave data. *Phys. Rev. D*, 76:081502(R), 2007.
- [22] B. Haskell, N. Andersson, D. I. Jones, and L. Samuelsson. Are neutron stars with crystalline color-superconducting cores relevant for the LIGO experiment? *Phys. Rev. Lett.*, 99:231101, 2007.
- [23] Curt Cutler. Gravitational waves from neutron stars with large toroidal B-fields. *Phys. Rev. D*, 66:084025, 2002.
- [24] B. J. Owen. Detectability of periodic gravitational waves by initial interferometers. *Class. Quant. Grav.*, 23:S1, 2006.
- [25] B. Haskell, L. Samuelsson, K. Glampedakis, and N. Andersson. Modelling magnetically deformed neutron stars. *Mon. Not. R. Astron. Soc.*, 385:531, 2008. (Preprint 0705.1780v1).
- [26] T. Akgün and I. Wasserman. Toroidal magnetic fields in type II superconducting neutron stars. *Mon. Not. R. Astron. Soc.*, 383:1551, 2008.
- [27] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs. The Australia Telescope National Facility Pulsar Catalogue. *Astron. J.*, 129:1993, 2005.
- [28] Available at <http://www.lsc-group.phys.uwm.edu/daswg/projects/lal.html>.
- [29] Curt Cutler and Bernard F. Schutz. The generalized F-statistic: Multiple detectors and multiple GW pulsars. *Phys. Rev. D*, 72:063006, 2005.
- [30] J. H. Conway and N. J. A. Sloane. *Sphere Packings, Lattices and Groups*. Springer-Verlag, 1988.
- [31] Benjamin J. Owen et al. Gravitational waves from hot young rapidly rotating neutron stars. *Phys. Rev. D*, 58:084020, 1998.
- [32] Benjamin J. Owen. Search templates for gravitational waves from inspiraling binaries: Choice of template spacing. *Phys. Rev. D*, 53:6749, 1996.
- [33] D. M. Whitbeck. *Observational consequences of gravitational wave emission from spinning compact sources*. PhD thesis, The Pennsylvania State University, 2006.
- [34] Reinhard Prix. Template-based searches for gravitational waves: efficient lattice covering of flat parameter spaces. *Class. Quant. Grav.*, 24:S481, 2007.
- [35] Australian Partnership for Advanced Computing, <http://nf.apac.edu.au/>.
- [36] Albert Lazzarini and Rainer Weiss. LIGO Science Requirements Document. LIGO technical document LIGO-E950018-02-E, 1996. <http://www.ligo.caltech.edu/docs/E/E950018-02.pdf>.